Analytical approach for investigating static response of laminated composite, FGM, FG-CNT cylindrical shells using various higher-order shear deformation theories

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Abstract

Composite structures are widely used for many engineering applications, such as aerospace and marine structures, energy and civil engineering, automobile, and petrochemical industries … A great number of studies on analyzing composite structures in literature have been published. However, developing mathematical models and computational programs for static analyses of laminated composite, FGM, and FG-CNT cylindrical shells subjected to different types of thermo-mechanical loads with various boundary conditions using higher-order shear deformation theories (HSDT) and analytical approaches may have many complex effects. In this work, the governing equations are derived from the different HSDTs including transverse shear and normal stresses. To obtain the static solution, these governing equations are solved by implementing simple trigonometric series and the Laplace transform. The transverse stress components are recovered by integrating the equation of equilibrium according to the 3D theory of elasticity. The present study accentuates the necessity of including transverse normal strain in the theory for composite shells.

Keywords: Analytical approach, higher-order shear deformation theory, composite cylindrical shell, various boundary conditions, thermo-mechanical load

1. Introduction

Thin-walled structures such as plates and shells are widely used in engineering applications such as civil engineering, automotive, aerospace, marine and naval engineering, nuclear power plants, etc. due to their multiple advantages such as high strength, lightweight, durability, etc. [1-2]. Nowadays, the advancement in material manufacturing leads to more and more practical applications of composite materials. Laminated composite, FGM, FG-CNT have great potential applications in many industrial areas thanks to their exceptional mechanical, electrical, and thermal properties [3-5]. Therefore, deformation and thermal stresses arising from the inhomogeneity of composite materials are important structural factors that are required to be considered in structure designing processes. Due to their simple mathematical models, the classical and the first-order shear-deformation theories are normally used in the structural analyses of plates and shells under mechanical and thermal loads [3, 6, 7].

In order to obtain more accurate results for analyzing the mechanical behaviors of composite plates and shells, the higher-order shear deformation theories (HSDTs) were developed as advancements of the first-order shear-deformation theories. Several types of HSDT widely used in modeling and analysis of thin-walled structures have been presented by Gupta and Talha in

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[8]. Using third-order shear deformation theory including transverse shear and normal stresses and transverse stress recovery technique, Doan et al. studied the stress concentration phenomenon of laminated composite, FGM, and FG-CNT cylindrical shells subjected to uniform mechanical load [9,10, 11]. These studies also confirmed the necessity of using HSDT, including transverse stress and normal stress (quasi-3D HSDT), to analyze the stress state of the structure in the boundary zones.

In this paper, several types of quasi-3D HSDT are employed to study static response of laminated composite, FGM, FG-CNT cylindrical shells. The governing equations and the boundary conditions are derived using the principle of virtual work. To obtain the static solution, these governing equations are solved by implementing simple trigonometric series and the Laplace transform. The transverse stress components are recovered by integrating the equation of equilibrium according to the 3D theory of elasticity. The stress distribution in the boundary zones is proposed for laminated composite, FGM, FG-CNT cylindrical shells subjected to thermal, and mechanical loads.

2. Theoretical formulation

In this study, we consider a laminated composite circular cylindrical shell of length *L*, radius *R*, and thickness *h* in the orthogonal curvilinear coordinate system $O\xi\theta z$ as shown in Fig. 1. For laminated composite shells, the structure includes *NL* layers, each layer is a homogeneous fiber-reinforced composite. The main direction of fiber reinforcement of each layer coincides with the direction of the local coordinate. The angle between the direction of fiber reinforcement and the vertical axis $O\xi$ of the general coordinate system is labeled as β.

For FGM shells, the mechanical and physical properties of inhomogenous and isotropic material are supposed to be a power function of a dimensionless radial coordinate:

$$P_{FGM}(z) = P_i V_{mat}, \quad (1)$$

where $V_{mat} = \left(r/r_i\right)^{\eta}$, P_{FGM} denotes material parameters, such as the Young's modulus *E*, the Poisson's ratio ν, the thermal expansion coefficient α and thermal conductivity coefficient κ; P_i is the parameter at the inner surface of the shell, respectively; η is the power-law index that is a real number; $r_i = R - h/2$ is the radius of the inner surface; *r* is radius of the considered point. The FGM material properties vary smoothly across the shell thickness.

For FG-CNT cylindrical shells, the CNTs volume fraction is uniform with the thickness direction. According to the rule of mixture and considering the CNT efficiency parameters, the effective mechanical properties of FG-CNTRC can be written as follows

$$\begin{aligned} &E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E_m, \\ &\frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E_m}, \ \frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G_m}, \end{aligned} \quad (2)$$

where $\eta_i\left(i=1,2,3\right)$ are CNT efficiency parameters, $E_{11}^{CNT}, E_{22}^{CNT}, G_{12}^{CNT}, V_{CNT}, E_m, G_m, V_m$ are Young's modulus, shear modulus, volume fraction of carbon nanotube and matrix, respectively. The relation between the CNT and matrix volume fractions is given as

$$V_{CNT} + V_m = 1, \ V_{CNT} = \frac{W_{CNT}}{W_{CNT} + \frac{\rho_{CNT}}{\rho_m} - \frac{\rho_{CNT}}{\rho_m} W_{CNT}} \quad (3)$$

in which ρ_m is density fraction of polymer matrix; W_{CNT}, ρ_{CNT} is mass fraction and density fraction of CNT, respectively.

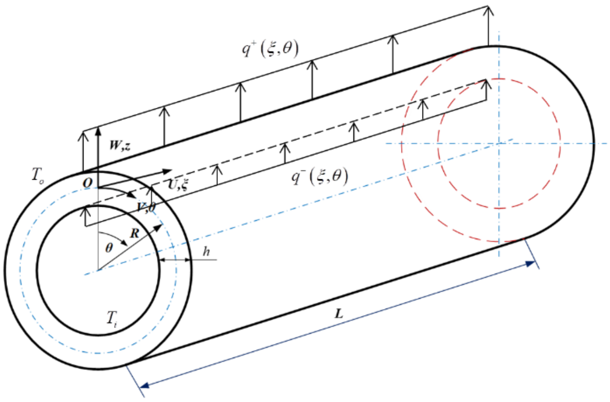


Fig. 1 Geometry and notations of a cylindrical shell

The displacement field of the shell in the orthogonal curvilinear coordinate system $O\xi\theta z$ is expressed by

$$\begin{aligned} u(\xi,\theta,z) &= \sum_{k=0}^{K} u_k(\xi,\theta)\frac{z^k}{k!}, \\ v(\xi,\theta,z) &= \sum_{k=0}^{K} v_k(\xi,\theta)\frac{z^k}{k!}, \\ w(\xi,\theta,z) &= \sum_{k=0}^{K-1} w_k(\xi,\theta)\frac{z^k}{k!}. \end{aligned} \quad (4)$$

where K is the order of shell theory. $u(\xi,\theta,z)$, $v(\xi,\theta,z)$ and $w(\xi,\theta,z)$ are 3D displacement components of an arbitrary point $P(\xi,\theta,z)$ at the distance z from the middle surface ($z=0$) according to the coordinate axes.

The linear strain-displacements relations are defined as follows:

$$\begin{aligned}&\varepsilon_\xi=\frac{1}{R}\frac{\partial u}{\partial \xi},\varepsilon_\theta=\frac{1}{R+z}\left(\frac{\partial v}{\partial \theta}+w\right),\\ &\gamma_{\xi\theta}=\frac{1}{R}\frac{\partial v}{\partial \xi}+\frac{1}{R+z}\frac{\partial u}{\partial \theta},\gamma_{\xi z}=\frac{1}{R}\frac{\partial w}{\partial \xi}+\frac{\partial u}{\partial z},\\ &\gamma_{\theta z}=\frac{1}{R+z}\frac{\partial w}{\partial \theta}+\frac{\partial v}{\partial z}-\frac{v}{R+z},\varepsilon_z=\frac{\partial w}{\partial z}.\end{aligned} \tag{5}$$

The stress-strain relationships for k-th layer are obtained as

$$\{\sigma\}=\left[T^{(k)}\right]^t\left[C^{(k)}\right]\left[T^{(k)}\right]\{\varepsilon\} \tag{6}$$

in which, $\{\sigma\}=\{\sigma_\xi\ \sigma_\theta\ \sigma_z\ \tau_{\xi\theta}\ \tau_{\xi z}\ \tau_{\theta z}\}^t$ is the stress vector in the general coordinate system; $\{\varepsilon\}=\{\varepsilon_\xi\ \varepsilon_\theta\ \varepsilon_z\ \gamma_{\xi\theta}\ \gamma_{\xi z}\ \gamma_{\theta z}\}^t$ is the strain vector in the general coordinate system, the transformation matrix $\left[T^{(k)}\right]$ of the k-th layer is defined as follow

$$\left[T^{(k)}\right]=\begin{bmatrix} c^2 & s^2 & 0 & 0 & 0 & cs\\ s^2 & c^2 & 0 & 0 & 0 & -cs\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & c & -s & 0\\ 0 & 0 & 0 & s & c & 0\\ -2cs & 2cs & 0 & 0 & 0 & c^2-s^2\end{bmatrix}, \tag{7}$$

$c=\cos\beta^{(k)}, s=\sin\beta^{(k)}$.

For FGM and FG-CNT, the angle $\beta^{(k)}$ is equal to zero. Matrix $\left[C^{(k)}\right]$ is derived as shown below

$$\left[C^{(k)}\right]=\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}^{(k)}, \tag{8}$$

where

$$C_{12}^{(k)}=E_1^{(k)}\left(\nu_{21}^{(k)}+\nu_{31}^{(k)}\nu_{23}^{(k)}\right)/\nu^{(k)}, C_{11}^{(k)}=E_1^{(k)}\left(1-\nu_{23}^{(k)}\nu_{32}^{(k)}\right)/\nu^{(k)}$$

$$C_{13}^{(k)}=E_1^{(k)}\left(\nu_{31}^{(k)}+\nu_{21}^{(k)}\nu_{32}^{(k)}\right)/\nu^{(k)}, C_{22}^{(k)}=E_2^{(k)}\left(1-\nu_{13}^{(k)}\nu_{31}^{(k)}\right)/\nu^{(k)}$$

$$C_{23}^{(k)}=E_2^{(k)}\left(\nu_{32}^{(k)}+\nu_{12}^{(k)}\nu_{31}^{(k)}\right)/\nu^{(k)}, C_{33}^{(k)}=E_3^{(k)}\left(1-\nu_{12}^{(k)}\nu_{21}^{(k)}\right)/\nu^{(k)}$$

$$C_{44}^{(k)}=G_{12}^{(k)}, C_{55}^{(k)}=G_{13}^{(k)}, C_{66}^{(k)}=G_{23}^{(k)},$$

$$\nu^{(k)}=\left(1-\nu_{12}^{(k)}\nu_{21}^{(k)}-\nu_{23}^{(k)}\nu_{32}^{(k)}-\nu_{13}^{(k)}\nu_{31}^{(k)}-2\nu_{13}^{(k)}\nu_{32}^{(k)}\nu_{21}^{(k)}\right).$$

It is assumed that the thermal load is axially symmetrical and varies only in the direction of thickness (radial orientation). The temperature distribution in the shell is expressed as [12]

$$-\frac{d}{dz}\left[\kappa\frac{dT}{dz}\right]=0. \tag{9}$$

The temperature boundary conditions are:

- At the inner surface ($z=-h/2$): $T=T_i$,
- At the outer surface ($z=h/2$): $T=T_o$.

The governing equations are based on the equilibrium state and derived by the use of the principle of virtual work.

$$\begin{aligned}&\frac{\partial N_\xi^{(0)}}{\partial \xi}+\frac{\partial N_{\theta\xi}^{(0)}}{\partial \theta}=0,\frac{\partial N_{\xi\theta}^{(0)}}{\partial \xi}+\frac{\partial N_\xi^{(0)}}{\partial \theta}+\partial N_{\theta z}^{(0)}=0,\\ &\frac{\partial N_{\xi z}^{(0)}}{\partial \xi}+\frac{\partial N_{\theta z}^{(0)}}{\partial \theta}-N_\theta^{(0)}-Rp_0=0,\\ &\frac{\partial N_\xi^{(i)}}{\partial \xi}+\frac{\partial N_{\theta\xi}^{(i)}}{\partial \theta}-RN_{\xi z}^{(i-1)}=0,\ i=1,2,...,\ K,\\ &\frac{\partial N_{\xi\theta}^{(i)}}{\partial \xi}+\frac{\partial N_\theta^{(i)}}{\partial \theta}-RN_{\theta z}^{(i-1)}=0,\ i=1,2,...,\ K,\\ &\frac{\partial N_{\xi z}^{(j)}}{\partial \xi}+\frac{\partial N_{\theta z}^{(j)}}{\partial \theta}-N_\theta^{(j)}-RN_z^{(j-1)}-Rp_j=0,\\ &j=1,2,...,(K-1),\end{aligned} \tag{10}$$

where the components of the stress-resultants in equations (10) are defined as:

$$\begin{aligned}&N_\xi^{(i)}=\sum_{k=1}^{NL}\int_{h_{k-1}}^{h_k}\sigma_\xi^{(k)}\left(1+\frac{z}{R}\right)\frac{z^i}{i!}dz, N_{\theta\xi}^{(i)}=\sum_{k=1}^{NL}\int_{h_{k-1}}^{h_k}\tau_{\xi\theta}^{(k)}\frac{z^i}{i!}dz,\\ &N_{\xi\theta}^{(i)}=\sum_{k=1}^{NL}\int_{h_{k-1}}^{h_k}\tau_{\xi\theta}^{(k)}\left(1+\frac{z}{R}\right)\frac{z^i}{i!}dz, N_\theta^{(i)}=\sum_{k=1}^{NL}\int_{h_{k-1}}^{h_k}\sigma_\theta^{(k)}\frac{z^i}{i!}dz,\\ &N_{\theta z}^{(i)}=\sum_{k=1}^{NL}\int_{h_{k-1}}^{h_k}\tau_{\theta z}^{(k)}\frac{z^i}{i!}dz, N_{\xi z}^{(i)}=\sum_{k=1}^{NL}\int_{h_{k-1}}^{h_k}\tau_{\xi z}^{(k)}\left(1+\frac{z}{R}\right)\frac{z^i}{i!}dz,\\ &N_z^{(i)}=\sum_{k=1}^{NL}\int_{h_{k-1}}^{h_k}\sigma_z^{(k)}\left(1+\frac{z}{R}\right)\frac{z^i}{i!}dz, p_i=q\left(1+\frac{h}{2R}\right)\frac{(h/2)^i}{i!}.\end{aligned} \tag{11}$$

For closed cylindrical shells, the boundary conditions for simply supported (S), clamped (C), and free (F) edges at $\xi = 0$ and $\xi = L/R$ are given by

$$C: \ u_i = v_i = w_j = 0,$$
$$F: N_{\xi}^{(i)} = N_{\xi\theta}^{(i)} = N_{\xi z}^{(j)} = 0, \tag{12}$$
$$i = 0,1,...,K; \ j = 0,1,...,(K-1).$$

3. Methodology of determining stresses

The solutions of equations (10) and boundary conditions (12) can be found using simple trigonometric series and the Laplace transform [9, 10]. The displacement components and the loads are represented by single trigonometric series as follows:

$$\begin{aligned}
u_i(\xi,\theta) &= U_{i0}(\xi) + \sum_{m=1}^{\infty} U_{im}(\xi)\sin m\theta, \\
v_i(\xi,\theta) &= V_{i0}(\xi) - \sum_{m=1}^{\infty} V_{im}(\xi)\cos m\theta, \\
w_j(\xi,\theta) &= W_{j0}(\xi) + \sum_{m=1}^{\infty} W_{jm}(\xi)\sin m\theta, \\
q^{\pm}(\xi,\theta) &= Q_0^{\pm}(\xi) + \sum_{m=1}^{\infty} Q_m^{\pm}(\xi)\sin m\theta, \\
T_i &= T_{i0}(\xi) + \sum_{m=1}^{\infty} T_{im}(\xi)\sin m\theta, \\
T_o &= T_{o0}(\xi) + \sum_{m=1}^{\infty} T_{om}(\xi)\sin m\theta.
\end{aligned} \tag{13}$$

The transverse stress components are calculated by integrating the equation of equilibrium according to the 3D theory of elasticity as bellows:

$$\begin{aligned}
\tau_{\xi z} &= -\frac{1}{R+z}\int_{-h/2}^{z}\left[\left(1+\frac{z}{R}\right)\frac{\partial \sigma_{\xi}}{\partial \xi} + \frac{\partial \tau_{\xi\theta}}{\partial \theta}\right]dz, \\
\tau_{\theta z} &= -\frac{R}{(R+z)^2}\int_{-h/2}^{z}\left[\left(1+\frac{z}{R}\right)\frac{\partial \sigma_{\theta}}{\partial \theta} + \left(1+\frac{z}{R}\right)^2\frac{\partial \tau_{\xi\theta}}{\partial \xi}\right]dz, \\
\sigma_z &= -\frac{1}{R+z}\int_{-h/2}^{z}\left[\left(1+\frac{z}{R}\right)\frac{\partial \tau_{\xi z}}{\partial \xi} + \frac{\partial \tau_{\theta z}}{\partial \theta} - \sigma_{\theta}\right]dz \\
&\quad + \frac{R-h/2}{R+z}q^{-}.
\end{aligned} \tag{14}$$

The above equations can guarantee the internal equilibrium state of the shell element, and at the same time, satisfy the boundary conditions on the inner and outer surfaces. The new representations of the transverse shear and normal stresses as above enable the stress-strain state of the shell to satisfy the equation of equilibrium according to the three-dimensional elasticity theory.

4. Numerical analysis and discussion

The first, the stress analysis of laminated composite cylindrical shells is conducted. The fully clamped shell has the following parameters: the radius R=0.1m, the relative length L/R=2, and the relative thickness R/h=20. We consider a laminated composite [0/90°/0] shell made from Graphite-Epoxy (AS/3501) [3]. The applied thermal load is due to a temperature distribution that varies in a linear in the thickness direction, the temperature rise at the outer surface is $\Delta T_o = 100°C$. The shell is subjected to the uniform pressure on the inner surface $q^{-} = 1MPa$. The non-dimensional stresses are defined as follows: $\left(\bar{\sigma}_{\xi}, \bar{\sigma}_{\theta}, \bar{\tau}_{\xi z}, \bar{\sigma}_z\right) = \frac{1}{\alpha_{eff}\Delta T_o E_{eff} R}\left(\sigma_{\xi}, \sigma_{\theta}, \tau_{\xi z}, \sigma_z\right)$.

in which

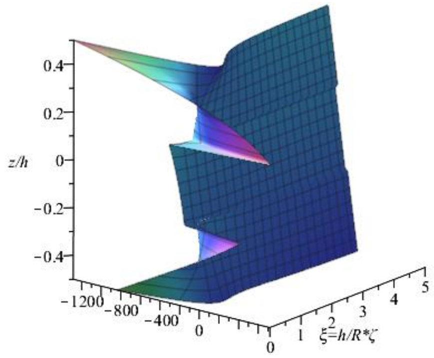
- $\alpha_{eff} = \alpha_1, E_{eff} = E_2$ for laminated composite shells.
- $\alpha_{eff} = \alpha_i, E_{eff} = E_i$ for FGM shells.
- $\alpha_{eff} = \alpha_m, E_{eff} = E_m$ for FG-CNTRC shells.

The distribution of non-dimensional stresses through the thickness of laminated composite shells in the boundary zone for the case of the theory K=3 is shown in Fig. 2.

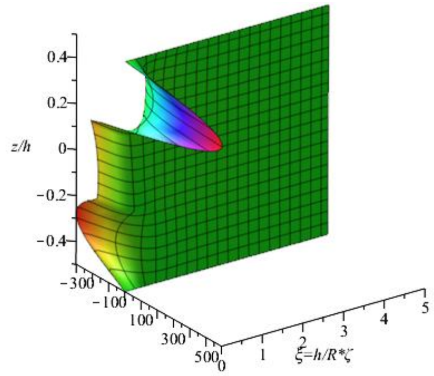
Second, we consider the thermomechanics responses of FGM cylindrical shells with the clamped-free boundary condition. The input parameters are following: the radius R=0.1m, the relative length L/R=1, and the relative thickness R/h=20; the parameters of the inner surface are $\nu_i = 0.3$, $E_i = 105.7\ GPa$, $\alpha_i = 6.9\times10^{-6}\ /\ ^oC$, $K_i = 18.1\ W/m.K$; the temperatures at the inner and outer surfaces are $\Delta T_i = 0\,^oC$, $\Delta T_o = 100\,^oC$, respectively; the shell is subjected to an inner pressure $q^{-} = 1MPa$. The power-law index η is equal 2. The distribution of non-dimensional stresses through the thickness of FGM shells in the boundary zone for the case of the theory K=4 is shown in Fig. 3.

For investigating the distribution of non-dimensional stresses through the thickness of FG-CNTRC cylindrical shells, the fully clamped shell with the following parameters: the relative length

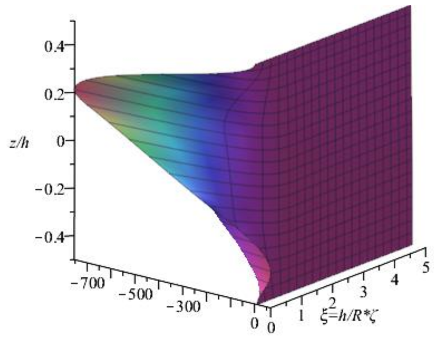
L/R=3, and the relative thickness R/h=10 is considered, the volume fraction $V_{CNT} = 0.28$.



a) The non-dimensional stress $\bar{\sigma}_{\xi}$

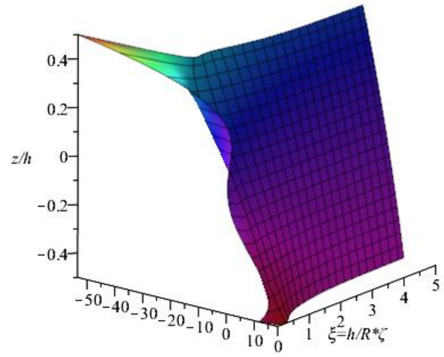


b) The non-dimensional stress $\bar{\tau}_{\xi z}$

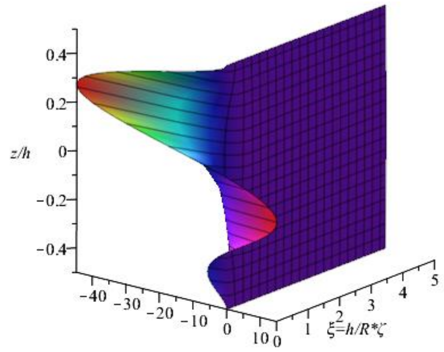


c) The non-dimensional stress $\bar{\sigma}_{z}$

Fig. 2 Distribution of non-dimensional stresses through the thickness for 3-layer symmetric [0/90°/0] cylindrical shell



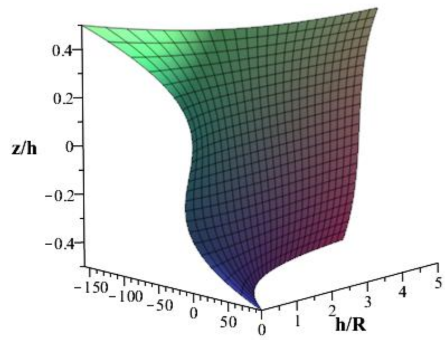
a) The non-dimensional stress $\bar{\sigma}_{\xi}$



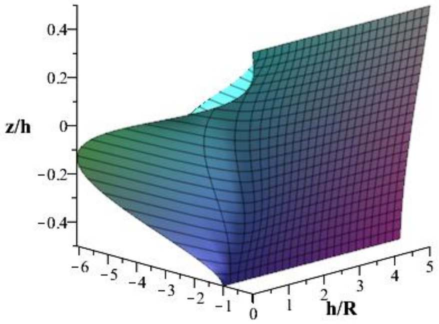
b) The non-dimensional stress $\bar{\sigma}_{z}$

Fig. 3 Distribution of non-dimensional stresses through the thickness for FGM cylindrical shell

The shell is subjected to the uniform pressure on the inner surface $q^{-} = 10MPa$; the temperatures at the inner and outer surfaces are $\Delta T_i = 200^oC$, $\Delta T_o = 0^oC$, respectively. The distribution of non-dimensional stresses through the thickness of FG-CNTRC shells in the boundary zone for the case of the theory K=3 is shown in Fig. 4.



a) The non-dimensional stress $\bar{\sigma}_{\xi}$



b) The non-dimensional stress $\bar{\sigma}_z$

Fig. 4 Distribution of non-dimensional stresses through the thickness for FG-CNTRC cylindrical shell

It can be seen from the Figs 2-4 that at a clamped edge, the stresses increase considerably. The shear $\bar{\tau}_{\xi z}$ and normal $\bar{\sigma}_z$ transverse stress must be included while investigating structures with a clamped-support boundary condition due to the existence of stress concentration. The size of the stress concentration zone is small, and does not exceed half of the thickness.

Conclusions

The application of higher-order shear-normal deformation theory and semi-analytical approach to analyze the thermoelastic response of laminated composite, FGM, FG-CNTRC cylindrical shells. The virtual work principle was used to derive the governing equations and boundary conditions within the framework of the several quasi-3D types of HSDT, which includes the effects of transverse shear and normal deformations. The governing equations for composite cylindrical shells with various boundary conditions are solved analytically using simple trigonometric series and the Laplace transform. Moreover, the transverse shear and normal stress are reconstructed using 3D linear elasticity equilibrium equations. Using higher-order shear-normal deformation theory, this article has focused on investigating the stress distribution in boundary zones. Due to boundary effects for composite shells, the stress concentration phenomenon also occurs at the clamped boundary zone. This phenomenon decreases very quickly with distance from the edge.

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